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CONDITIONS GOVERNING THE EXCITATION AND PROPAGATION OF SECOND SOUND

V. P. Peshkov
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The experiments of Kapitsa [1,2] and of Andronikashvili [3] and the experiments with second sound showed that the theory of superfluidity, assumed by Landau [5], in helium qualitatively approached actuality; quantitative discrepancies, however, were observed. Thus, the values ρ_n/ρ (the ratio of that part of density connected with thermal motion to the full density of helium II), obtained from experiments with second sound and from direct experiments on the "being-carried-along (entrainment)" of helium II by a pile of oscillating discs closely located, are in very close agreement. But the measurements of impulse relative to the reaction of heat flow on a small vane give quantities several times less than expected on the basis of the hydrodynamic part of Landau's theory.

The data obtained by Kessom, Saris and Meyer [6], and Meyer and Mellink [7] for heat-exchange in capillaries and slots also do not fit qualitatively into the framework of the hydrodynamic part of Landau's theory. Calculation of the basic constants of the microscopic (microcosmic) part of Landau's theory from the heat capacity C and entropy S and from the speed of second sound u_2 also lead to significantly different values. In a recent work of Landau [8], at the cost of introducing a third new constant, he succeeded in obtaining, in the interval of temperatures from 1.3 °K to 1.7—1.8°K, an agreement of theory with experimental data for C_1S_1 and u_2.

Since, however, there are already in the hydrodynamic part of the theory considerable discrepancies and nonalignments with experiments, it is impossible to assume that Landau's theory of superfluidity of helium II describes fully and satisfactorily all the basic properties of helium II. The recent works of Tisza also do not give the complete picture of helium II's properties and are a mixture of thermodynamic considerations with ideas from

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Landau's theory, not giving even an approximately correct, in a certain region of temperatures, empirical ratio ρ_n/ρ . Therefore, it seems expeditious in the discussion of second sound in helium II, when using the basic ideas of Landau, to employ whenever possible the well-known thermodynamic quantities and relations and to introduce only such new concepts, without which it would be impossible to clarify the properties of helium II and which flow directly from experiments.

Kapitsa, in studying the properties of helium II, discovered that the flow of heat flowing out of a capillary, in contrast with ordinary fluidity, propagates not uniformly to all sides, but possesses the character of an extremely set, directed flow or stream constant up to distances at least 30 times the diameter of the capillary. During this strongly directional streaming, the flow of heat exerts considerable pressure on any vane placed in the stream or current; that is, the heat in helium II possesses the property of inertia.

At the same time, Kapitsa revealed that helium II flowing through thin slots and capillaries does not experience any viscous forces and does not carry with itself any heat.

Andronikashvili established that a pile of disks closely located one next to another, performing torsional oscillations, "carries along" or "entrains" with itself not the whole mass of helium II, but only a part of it; during this "entrainment," the portion of the "entrainable" liquid, that is, that part able to be dragged along by the disks, varies with temperature, after reaching unity at the lambda point (λ -point).

By comparing the experiments summarized above, it is natural to assume, as even Landau did, that the heat links (takes up) with its motion not all the helium, but only a part of it, equal to the rho-ratio ρ_n/ρ , where ρ is the density of helium II and ρ_n (subscript "n" means "normal") is that part (portion) of the density linked or connected (taken up) with the thermal movement. The remaining part of the density ρ_s is that part of the superfluid helium such that: $\rho=\rho_n+\rho_s$ (subscript "s" means "superfluid"). Such a division of ρ into ρ_n and ρ_s does not at all imply that part of the atoms of helium constantly remains in the unexcited state, and part in the excited; it only permits one to draw up a more graphic picture for describing the properties of helium II.

In accordance with the experiments of Kapitsa, the difference in heat content Q between that superfluid part of helium II flowing through a capillary and helium II in the ordinary state is equal to $Q_1 = TS$, where T is the absolute temperature and S is the entropy. It is logical, therefore, to assume that all the heat is connected (bond) with only the normal part of helium II; therefore, during flow of the heat of density w, the velocity of motion of the normal part of helium II will equal:

$$v_n = w/\rho ST, \qquad (1)$$

and the impulse corresponding to heat flow is $\rho_n v_n$. The ordinary flow of helium II is defined as $j=\rho_n v_n+\rho_s v_s$ where v_s is the velocity of the superfluid part of helium II.

The impulse of heat flow or its property of inertia is a new physical concept, and the dimension of inertia of heat flow ρ_n is a new physical quantity. As for other assumptions made, the density of kinetic energy in helium II is defined, as shown in Lifshits' work (10), in the form of a sum thus:

$$\varepsilon = \tfrac{1}{2}\rho_n v_n^2 + \tfrac{1}{2}\rho_s v_s^2, \qquad (2)$$

that is, energy does not equal zero $\varepsilon \neq 0$ when the current may be zero: $j = 0$; non-zero energy exists during the presence of heat flow.

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An extremely small coefficient of thermal expansion alpha α also appears to be a peculiarity of helium II: $\rho^{-1}(\partial\rho/\partial T) = \alpha = 10^{-2}$ / degree, which fact leads practically to a nondependence between mechanical and thermal motion; that is, an increase in temperature almost does not cause any increase in pressure, and vice versa. This also leads to the fact that, with accuracy up to 1—2 percent, it is possible to set up the following relations:

$$(\partial S/\partial T)_p \cong (\partial S/\partial T)_v \equiv C/T, \qquad (3)$$

$$(\partial p/\partial \rho)_S \cong (\partial p/\partial \rho)_T \doteqdot \partial p/\partial \rho. \qquad (4)$$

This last case permits one to solve the problem relating to the propagation of ordinary and second sound independently. Actually, from the equations stating the law of conservation of mass

$$\partial\rho/\partial t + \operatorname{div} j = 0, \qquad (5)$$

and Newton's law

$$\partial j/\partial t + \nabla p = 0, \qquad (6)$$

and after substituting $\nabla p = (\partial p/\partial\rho)\nabla\rho$ and eliminating, by differentiation, ordinary flow j, we then have:

$$\partial^2\rho/\partial t^2 = (\partial p/\partial\rho)\Delta\rho, \qquad (6')$$

that is, we then have the equation expressing the propagation of ordinary sound with the velocity: $u_1 = (\partial p/\partial\rho)^{1/2}$.

In order to solve the problem relating to second sound, we shall employ the law of conservation of heat:

$$\rho C(\partial T/\partial t) + \operatorname{div} w = 0, \qquad (7)$$

which can be described in such a simple form as this, by taking the relations (3) into account. Further, let us employ the peculiar property of inertia of heat flow in helium II. Let us assume, on the analogy of Gogate and Pathak's formulation [11], that the vibrations of second sound progress reversibly; that is, strictly obey the second law of thermodynamics:

$$dA/W = dT/T \qquad (8)$$

where dA is the work contained on account of the difference of temperature dT during the transfer of a quantity of heat W at a temperature T. Let us choose a layer dx in which the temperature varies by dT; then through a unit surface in time dt there will issue a quantity of heat $W = w_x dt$ which causes a variation in the kinetic energy of a unit surface of a layer dx, in the amount $dA = dx\,d\varepsilon$ and in accordance with (8) $dx\,d\varepsilon/w_x dt = -dT/T$ or

$$T(\partial\varepsilon/\partial t) = -w\nabla T; \qquad (9)$$

the minus sign arises from the face that for any w_x then $d\varepsilon > 0$ is positive only when $\partial T/\partial x < 0$.

We notice that in equation (7) it would have been necessary to add a term $\partial\varepsilon/\partial t$ indicating the kinetic energy of heat flow which is converted during vibrations into thermal energy, and back; however, this term will be of the second order of smallness relative to the basic vibration and we can neglect it here, as well as all other terms of higher orders of smallness.

Further, by utilizing the fact that, because of the smallness of the coefficient of thermal expansion during thermal oscillations of j, it is possible to set it equal to zero, we then have: $\varepsilon = \rho_n v_n^2/2 + \rho_n^2 v_n^2/2\rho_s = \rho\rho_n v_n^2/2\rho_s$ or by substituting v_n according to formula (1):

$$\varepsilon = \rho_n w^2/2\rho\rho_s S^2 T^2. \qquad (10)$$

By substituting $d\varepsilon = \rho_n w\, dw/\rho\rho_s S^2 T^2$ in (9), we then have:

$$\partial w/\partial t = -(\rho\rho_s S^2 T/\rho_n)\nabla T. \qquad (11)$$

By eliminating, by means of differentiation, w from (7) and (11), we then obtain

$$\ddot{T} = (\rho_s S^2 T/\rho_n C)\Delta T. \qquad (12)$$

The last equation represents the law stating the propagation of second sound with the velocity:

$$u_2 = (\rho_s S^2 T/\rho_n C)^{1/2} \qquad (13)$$

It is worth noting that representation of the density of kinetic energy during thermal motion in the form (2) and relation (1) do not appear possible together. It is fully permissible to represent energy in the following form:

$$\varepsilon = \mu w^2/2, \qquad (2a)$$

thus eliminating the artificial division of helium II into a superfluid and a normal part.

In the last case, formula (9) and $d\varepsilon = \mu w\, dw$ give us $\partial w/\partial t = -(1/\mu T)\nabla T$ and further with (7) $\ddot{T} = (1/\rho\mu CT)\Delta T$; that is

$$u_2 = (\rho\mu CT)^{-1/2}. \qquad (13a)$$

Comparing (13) and (13a), we concluded that $\mu = \rho_n/\rho\rho_s S^2 T^2$.

As is obvious, it is possible to determine from experiments with second sound the values of μ or ρ_n; the problem, however, concerning the impulse of heat flow remains open; that is, expression (1) cannot be verified (proved) by experiments with second sound. That the question concerning the experimental verification (test or check) of relation (1) is not a trivial one is obvious from the experiments of Kapitsa [1], where the values for impulse turn out to be considerably less (approximately two times less) than expected on the basis of formula (1).

As is well known, by solving the equation $\ddot{T} = u_2^2 \Delta T$ one obtains functions of the form $f(t \mp x/u_2)$. Therefore, for waves propagated with the velocity u_2, we have $\partial T/\partial t = -u_2 \Delta T$. By using formula (7), we obtain $Cu_2 \Delta T = \operatorname{div} w$ or $\operatorname{div}(\rho C T u_2) = \operatorname{div} w$, where ρ, C and u_2 are constants. If we denote the variable part of temperature T by T', then from the preceding equation there results a relation between the oscillation of heat-flow and that of temperature in a traveling wave of second sound:

$$\rho C T' u_2 = w. \qquad (14)$$

This formula establishes the connection not only between amplitudes, but also between the phases of oscillations of temperature and heat-flow.

The density of kinetic energy may be written down, with the use of formulas (10) and (13) [or, for another variation, (2a) and (13a)], in the form:

$$\varepsilon = w^2/2\rho C T u_2^2 . \qquad (15)$$

By substituting w with T' according to formula (14), we obtain:

$$\varepsilon_\pi = \rho C T'^2/2T. \qquad (16)$$

The epsilon sub-pi (ε_π) represents the density of potential energy, which during the wave processes everywhere equals the density of kinetic energy (sub-pi in ε_π denotes "potential"). If by T_0' and w_0 we denote the amplitudes of oscillations of temperature and heat-flow, then the average flow, with respect to time, of the energy of second sound is:

$$q = 1/2(\varepsilon_0 + \varepsilon_{\pi 0}) u_2 = w_0^2 u_2/2\rho C T u_2^2 = \rho C T'^2_0 u_2/2T \qquad (17)$$

or by substituting w_0 or T_0' according to formula (14), we obtain:

$$q = T_0' w_0/2T \qquad (17')$$

This expression in second sound appears analogous to Poynting's vector in electromagnetic vibrations.

We now proceed to the conditions governing the excitation and propagation of second sound in helium II. Lifshits discussed several methods of radiating second sound; however, they were all hardly applicable in practice. The most effective method among them represented radiation from a surface with a periodically varying temperature; that is, with conditions at the boundary represented by the relation: $T' = T_0' e^{i\omega t}$.
For high temperatures and radiation of sound in a gas, the thermal capacity of metals is found to be considerably greater than the thermal capacity of gases; therefore, this formula of Lifshits does not represent the work of forming a surface with periodically varying temperature. Wente [12] actually constructed such a "thermophone" and carried out tests on it. For low temperatures the thermal capacity of helium II is considerably greater than the thermal capacity of metals, and the formation of a surface with a given (fixed) variation of temperature during arbitrary heat-flow seems to be an extremely complicated problem to solve.

In the works of the author [13], there were two methods used to radiate second sound: thermal method and filtration method. The thermal method represents the radiation of second sound by a variable flow of heat which is generated by a heat-source without inertia during through-passage by means of its variable current (flow); that is, with the condition at the boundary (for $x = 0$):

$$w = w_0 e^{i\omega t}, \quad j = 0. \qquad (18)$$

The steady heat-flow resulting for this condition, in view of the linearity of the equations, does not influence the propagation of second sound, and the quadratic (second-power) effects turn out to be insignificant. The condition $j = 0$ and the equating to zero of the coefficient of thermal expansion leads to the excitation of only second sound, the traveling wave of which will have the following form:

$$w = w_0 e^{i\omega(t - x/u_2)}, \quad T' = T_0' e^{i\omega(t - x/u_2)} \qquad (19)$$

where, in accordance with (14) $T_0' = w_0/\rho C u_2$.

The second method relating to the radiation second sound consists of the periodic "punching" of helium II through a filter. If the filter consists of very fine pores, then only the superfluid part of helium II will pass through these pores; that is, at the boundary ($x=0$):

$$j=j_0 e^{i\omega t}, \quad w=0. \qquad (20)$$

With such a boundary condition it is impossible to satisfy separately both the wave of ordinary sound and the wave of second sound, because for second sound $j=0$ and, in ordinary sound, helium oscillates as a whole and $w=jST$; therefore, the solution is successfully found in the form of two waves, namely ordinary and second sounds:

$$j_1=j_0 e^{i\omega(t-x/u_1)}, \quad w_2=w_{20} e^{i\omega(t-x/u_2)}. \qquad (21)$$

These two waves satisfy the boundary conditions for $x=0$.

$$w_{20}=-j_0 ST, \quad T'_{20}=-j_0 ST/\rho C u_2, \quad j_{10}=j_0. \qquad (22)$$

The intensity of oscillations of the second sound, in accordance with (17), will be $q_2=S^2Tj_0^2/2\rho C u_2$, and the intensity of ordinary sound will be:

$$q_1=j_0^2 u_1/2\rho$$

The ratio of these two intensities are:

$$q_2/q_1=S^2T/Cu_1u_2. \qquad (23)$$

At 2°K, this quantity amounts to 0.1, but at lower temperatures it is still less. Thus, the intensity of ordinary sound during excitation by the filtration method is considerably greater than the intensity of the simultaneously radiated second sound.

The amplitudes of oscillations of pressure and density in ordinary sound is determined according to formulas (5) and (6):

$$p'_{10}=u_1 j_0, \quad \rho'_0=j_0/u_1. \qquad (24)$$

The oscillations of temperature in ordinary sound will be caused only on account of adiabatic compression (condensation) and expansion; that is, $T'_{10}=(\partial T/\partial p)_S p'_{10}$. According to the well-known thermodynamic equality, we have:

$$\left(\frac{\partial T}{\partial p}\right)_S=-\frac{(\partial S/\partial p)_T}{(\partial S/\partial T)_p}=\frac{T}{C}\left(\frac{\partial V}{\partial T}\right)_p=\frac{\alpha T}{\rho C},$$

where α is the coefficient of heat expansion; therefore, the amplitude of oscillations of temperature in ordinary sound is:

$$T'_{10}=(\alpha T/\rho C)p'_{10}=(\alpha u_1 T/\rho C)j_0. \qquad (25)$$

The ratio of the amplitudes of oscillations of temperature in second sound and in ordinary sound equals: $T'_{20}/T'_{10}=S/\alpha u_1 u_2$. At 2°K this quantity is of the order 20, and at 1.6°K it is around 10; that is, the observation with respect to oscillations of temperature is considerably more favorable for second sound in comparison with ordinary sound.

The oscillations of pressure and density in second sound is caused only on account of the heat-source and cold-sink. To determine ρ'_2 and p'_2 let us use equations (5) and (6) and the property: $\partial/\partial t=-u_2^{-1}\nabla$ true for all quantities in waves of second sound; then $\rho'_2=j_2/u_2$, and $j_2=p'_2/u_2$; that is, $\rho'_2=p'_2/u_2^2$. Further representing ρ'_2 in the variables p'_2 and T'_2, we have: $\rho'_2=(\partial\rho/\partial p)_T p'_2+(\partial\rho/\partial T)_p T'_2=p'_2 u_1^{-2}-\alpha\rho T'_2$. Substituting here ρ'_2, we have:

$$p'_2=-\frac{\alpha\rho u_1^2 u_2^2}{u_1^2-u_2^2}T'_2\approx-\alpha\rho u_2^2 T'_2, \quad \rho'_2=-\alpha\rho T'_2. \qquad (24a)$$

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(A more exact relation between the amplitudes of oscillations of density, pressure and temperature in second and ordinary sounds is introduced in the work of Lifshits [10].) Comparing (24a) and (24), we obtain for the ratio of amplitudes of oscillations of pressures and densities in second and ordinary sounds, during radiation by the method of filtration, the following expression:

$$p'_{20}/p'_{10} = \alpha u_2 ST/u_1 C, \quad \rho'_{20}/\rho'_{10} = \alpha u_1 ST/u_2 C. \tag{26}$$

For $2^\circ K$, $p'_{20}/p'_{10} \approx 3\cdot 10^{-4}$ and $\rho'_{20}/\rho'_{10} \approx 3\cdot 10^{-2}$; that is, oscillations corresponding to second sound almost completely mask ordinary sound.

Now let us study the case of standing waves in a cylindrical pipe, one end of which is covered by a flat oscillator and the other end is covered with a flat reflector. Let us assume that the attentuation (dying-out or extinguishment) per unit length of the tube equals gamma γ; that is, the solution of the sonic problem will be of the form $w \exp\{i\omega t \pm (\gamma + i\omega/u_2)x\}$. If the radiation is induced by the thermal method, then it is possible to consider that only second sound is propagated. Then the boundary conditions may be written in the form:

$$j = 0,\ w = w_0 \text{ for } x = 0;\ j = 0,\ w = 0 \text{ for } x = l$$

(where l is the length of the tube)

The solution is obtained as the sum of two waves traveling in opposite directions:

$$w = w_1 e^{i\omega t - (\gamma + i\omega/u_2)x} + w_2 e^{i\omega t + (\gamma + i\omega/u_2)x}$$

therefore,

$$w_1 + w_2 = w_0,\quad w_1 e^{-(\gamma + i\omega/u_2)l} + w_2 e^{(\gamma + i\omega/u_2)l} = 0 \tag{27}$$

Hence:

$$w_1 = w_0/[1 - e^{-2(\gamma + i\omega/u_2)l}]. \tag{28}$$

From the last formula it is obvious that resonances will hold for $\omega l/u_2 = n\pi$ where $n = 1, 2, 3, \ldots$. If γl is small, then during resonance:

$$w_1 = w_0/(1 - e^{-2\gamma l}) \approx w_0/2\gamma l. \tag{29}$$

For small disturbances; that is, for $\omega l/u_2 = n\pi + \omega\Delta/u_2$ where $\omega\Delta/u_2$ is small, we have:

$$w_1 = \tfrac{1}{2} w_0 (\gamma l \pm i\omega\Delta/u_2)^{-1} = \tfrac{1}{2} w_0 e^{i\varphi}[(\gamma l)^2 + (\omega\Delta/u_2)^2]^{-1/2} \tag{30}$$

As is obvious from (27), the nodes of oscillations of heat-flow will be formed at the reflecting and radiating surfaces.

In order to determine the oscillations of temperature, it is necessary to take into consideration the fact that, for waves traveling in opposite directions, the oscillations of temperature will be of different signs, if the directions of heat-flow are the same; therefore, from (13) we have $T'_1 = w_1/\rho u_2 C$ and $T'_2 = -w_2/\rho u_2 C$ and also:

$$T' = T'_1 + T'_2 = (w_1 - w_2)/\rho u_2 C. \tag{31}$$

At the boundaries during small disturbances:

$$T' = (w_0/\rho u_2 C) e^{i\varphi}[(\gamma l)^2 + (\omega\Delta/u_2)^2]^{-1/2}. \tag{32}$$

Thus, at the reflecting and radiating surfaces there will be loops (antinodes) of temperature-oscillations and nodes of oscillations of flow.

If one disturbs the resonance volume, so that the amplitude of oscillations of temperature is decreased by $\sqrt{2}$, then

$$\gamma l = \omega \Delta_0 / u_2 , \tag{33}$$

that is, by such a method it is possible to measure the coefficient of attenuation (damping and drying out) of second sound. During resonance, we have $\Delta = 0$; therefore, from (32) and (33) we have;

$$T' = w_1 / \rho C \omega \Delta_0 . \tag{34}$$

In case of resonance during radiation by the filtration method, the matter becomes somewhat more complicated. Here it is necessary to consider both sounds. Analogously to traveling waves, the solution can be resolved into parts corresponding to ordinary sound and to second sound:

$$j_1 = j_{11} e^{i\omega t - (\gamma_1 + i\omega/u_1)x} + j_{12} e^{i\omega t + (\gamma_1 + i\omega/u_1)x},$$

$$w_2 = w_{21} e^{i\omega t - (\gamma_2 + i\omega/u_2)x} + w_{22} e^{i\omega t + (\gamma_2 + i\omega/u_2)x}.$$

The boundary conditions will be:

$$w = 0, \quad j = j_0 \quad \text{for } x = 0$$
$$w = 0, \quad j = 0 \quad \text{for } x = l$$

Therefore:

$$j_{11} + j_{12} = j_0 ; \quad j_{11} e^{-(\gamma_1 + i\omega/u_1)l} + j_{12} e^{(\gamma_1 + i\omega/u_1)l} = 0. \tag{35}$$

Taking, further, (21) and (22) into account, we have:

$$w_{21} + w_{22} = -j_0 ST, \quad w_{21} e^{-(\gamma_2 + i\omega/u_2)l} + w_{22} e^{(\gamma_2 + i\omega/u_2)l} = 0 \tag{36}$$

From the last equations there is obtained:

$$j_{11} = j_0 [1 - e^{-2(\gamma_1 + i\omega/u_1)l}]^{-1} \text{ and } w_{21} = -j_0 ST [1 - e^{-2(\gamma_2 + i\omega/u_2)l}]^{-1}.$$

In case of weak damping and small disturbance, we have the following equation:

$$j_{11} = \tfrac{1}{2} j_0 (\gamma_1 l_1 + i\omega\Delta_1/u_1)^{-1}; \text{ and } w_{21} = -\tfrac{1}{2} j_0 ST (\gamma_2 l_2 + i\omega\Delta_2/u_2)^{-1}.$$

Therefore, in the general case, resonance of ordinary and second sounds will exist for various l's. Taking (25) and (14) into consideration, we obtain for amplitudes of oscillations of temperature at the loops (antinodes), and consequently on the boundaries too:

$$T_1' = (\alpha u_1 T j_0 / \rho C) [(\gamma_1 l_1)^2 + (\omega \Delta_1 / u_1)^2]^{-1/2} \tag{37}$$

$$T_2' = (S T j_0 / \rho C u_2) [(\gamma_2 l_2 + (\omega \Delta_2 / u_2)^2]^{-1/2} \tag{38}$$

In this manner, during radiation by the filtration method and during measurement of temperature-oscillations, it is possible to observe the resonances of both ordinary sound and second sound.

During resonance there is always set up such an amplitude of oscillations for which the losses in the resonator are fully compensated by the energy of the emitter (radiator); therefore, it seems interesting to determine the quantity of energy entering the resonator.

In the case of the excitation of second sound by the thermal method, the flow of energy from each unit of surface of the heat-source on the basis of formula (17) is equal to $q = T_0' W / 2T$. For standing waves this flow is determined as the difference between the leaving (exit) and entering (init) energies; that is:

$$q = (T_{10}' w_{10} - T_{20}' w_{20}) / 2T = (w_{10}^2 - w_{20}^2) / 2 \rho u_2 C T$$

[by substituting T' according to formula (14)]. If we use the method of complex variables; that is, if we write $w = w_0 e^{i\varphi}$, then

$$w_{10}^2 - w_{20}^2 = Re (w_1 - w_2)(w_1^* + w_2^*).$$

Using (31) and the first of the boundary conditions (27), we obtain

$$q = Re\{T'w^*/2T\}. \qquad (39)$$

This is the most general expression for the flow of energy in second sound. It is noteworthy that this equation holds true also for ordinary heat waves.

In the preceding discussions, it was assumed that attenuation (damping) represents a volume effect. Under actual conditions, however, the attenuation (damping) is determined mainly by the thermal and viscous dispersion at the walls and by the dispersion through the slots at the ends of the resonator. By the expression γ^2 , however, we understand some sort of effective attenuation (damping), which decreases the amplitude of the wave returning to the emitter (radiator); therefore, it is possible to assume that the revealed formulas will be approximately correct. Then on the basis of formula (34), we will have:

$$qQ = w_0^2 Q/2\rho C T\omega\Delta_0 = \rho C T\omega\Delta_0 T_0'^2 Q/2T, \qquad (40)$$

where Q is the surface of the emitter (radiator). This expression determines the full power dischargeable in compensation for losses. The losses in the general case can be broken down into three parts: (1) volumes, losses; (2) losses on the surface; and (3) losses on the boundaries. The volume losses are fully determined by the properties of helium II and are characterized, as in ordinary sound, by a quadratic (second-power) dependence upon frequency. The surface losses consist of thermal and viscous losses. The thermal losses may be evaluated (calculated).

Since the thermal capacity of hard bodies at low temperatures are considerably less than the thermal capacity of helium II, then it is possible to assume that the amplitude of oscillations of temperature in helium of the boundary of a hard body is set (assigned) by th oscillations of temperature in helium; that is, by the oscillations of second sound. If $T = T'e^{i\omega t}$ on the boundary, then the problem of thermal conductivity within a hard body, in agreement with the equation $C_1(\partial T/\partial t) = \lambda(\partial^2 T/\partial z^2)$, leads to the solution in the form following: $T = T'\exp\{-(i\omega C_1/\lambda)^{1/2} z\}$, where C_1 is the volumetric thermal capacity (specific heat) and λ is the body's thermal conductivity. The flow of heat within the surface is determined thus:

$$w = -\lambda(\partial T/\partial z)_0 = T'\sqrt{i\omega\lambda C_1}.$$

Therefore on the basis of formula (59) for the absorbable energy, we obtain:

$$q_T = (T'^2/2T)(\lambda\omega C_1/2)^{1/2} \qquad (41)$$

Averaging ("neutralizing" or smoothing out) with respect to the whole surface for a sinusoidal distribution of the amplitude on the surface gives, as this holds true in the case of standing waves, the following expression:

$$q_T = (T_0'^2/4T)(\lambda\omega C_1/2)^{1/2} \qquad (42)$$

The surface losses caused by viscous forces (friction), which appear during the movement of heat in helium II along the walls, also can be calculated. True, here it is necessary to use the quantity of impulse of heat-flow; therefore, if it turns out that the expression for impulse of heat-flow is needed exactly, then the calculation given below demands correction. On the basis of Landau's theory, in connection with motion in helium II, only the normal part takes part (participates); therefore, the equation of viscous motion at the wall has the form:

$$\rho_n(\partial v_n/\partial t) = \eta(\partial^2 v_n/\partial z^2).$$

where the speed is directed along the surface, and z is the coordinate directed perpendicularly to the surface. For the case of sound where

$v_n = v_n e^{i\omega t}$ we have $i\omega\rho_n v_n = \eta\, \partial^2 v_n / \partial z^2$ hence;

$$v_n = v_{n0}\left(1 + e^{-(i\omega\rho_n/\eta)^{1/2}}\right)e^{i\omega t}. \qquad (43)$$

The depth of penetration (permeation or infiltration) of the viscous waves for $T = 1.3°K$ and for $\nu = 100$ cycle/sec is $\Lambda = (2\eta/\omega\rho_n)^{1/2} \sim 10^{-3}$ and for higher temperatures this depth is still less. Therefore, $r \gg \Lambda$ and it is possible, by neglecting (disregarding) the curving surface, to calculate the losses for the flat case by taking (assuming) for v_{n_0} the amplitude of oscillations of second sound. The calculation of losses for a distribution of velocities, in accordance with formula (43) for ordinary fluids, is well known (14) and leads, after averaging ("neutralizing" or smoothing-out) with respect to time, to the following value:

$$q_\eta = v_n^2 (\omega \rho_n \eta / 8)^{1/2} \qquad (44)$$

per each unit of surface. Then in the given case, instead of the full velocity and density, what enters (penetrates) is the velocity and density of just the normal part, characteristically for helium II, but it does not change the form of the formula. In standing waves, v_n varies along the resonator sinusoidally, with an amplitude of variation equal to $2 v_{n1}$; therefore, the averaging ("neutralizing" or smoothing-out) of over the length of the resonator leads to the quantity:

$$q_\eta = v_{n1}^2 (\omega \rho_n \eta / 2)^{1/2} \qquad (45)$$

per each unit of surface. Substituting w_1 from formula (1) for v_{n1} with the aid of formula (14) and taking into consideration that $T_0' = 2T_1'$, we then obtain for viscous losses per unit surface of the resonator the expression:

$$q_\eta = (T_0'^2 C^2 u_2^2 / 4S^2T^2)\, \omega\rho_n/2)^{1/2} \qquad (46)$$

Thus the general expression for losses can be written in the form:

$$Qq = Q T_0'^2 \frac{\rho C \omega \Delta_0}{2T} =$$

$$= \frac{Q T_0'^2}{2T}\left\{\rho C u_2 \gamma l + \frac{Q_1}{Q}\left(\frac{C^2 u_2^2}{2S^2 T}\sqrt{\frac{\omega\rho_n\eta}{2}} + \frac{1}{2}\sqrt{\frac{\lambda\omega C_1}{2}}\right) + \frac{2\varkappa T}{Q}\right\},$$

or after reducing it, we have:

$$\omega\Delta_0 = \gamma l + \frac{Q_1}{2Q}\left(\frac{C u_2^2}{\rho S^2 T}\sqrt{\frac{\omega\rho_n\eta}{2}} + \frac{1}{\rho C}\sqrt{\frac{\lambda\omega C_1}{2}}\right) + \frac{2\varkappa T}{\rho C Q} \qquad (47)$$

where Q_1 is the general surface on which the losses occur and kappa $\varkappa$ is the coefficient determining the boundary (rim) losses.

The final expression permits, with respect to the width of the resonance curve for various values of ω and l, one to determine the losses in second sound, and hence to determine separately the volumetric, surface, and boundary losses.

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